MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXII. JANUARY 10, 1902. No. 3

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Rev. Charles Lakeman Tweedale, M.A., The Vicarage, Weston, near Ottley,

were balloted for and duly elected Fellows of the Society.

The following Candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

John Waller Lawrence Child, Vernham, Merton Hall Road, Wimbledon (proposed by Col. E. E. Markwick) ;

José Comas Solá, S. Felipe, 29 S. Gervasio, Barcelona, Spain (proposed by A. Stanley Williams) ;

Rev. John Horsley Haslam, M.A. Cantab., Vicar of St. Saviour's, Denmark Park, S.E. (proposed by D. G. Simpson) ;

Joseph Lunt, B.Sc., F.I.C., Royal Observatory, Cape of Good Hope (proposed by Sir David Gill) ; and

Joges Chandra Ray, M.A., Professor of Physical Science, Cuttack, Bengal, India (proposed by U. Chandra Ghosh).

Seventy-six presents were announced as having been received since the last meeting, including amongst others :—

Galileo, Opere, Edizio Nazionale, vol. 11, presented by the Italian Government ; 22 Charts of the international photographic chart of the heavens, presented by the Paris Observatory ; Wedgwood Medallion of Sir Wm. Herschel by Flaxman, presented by Sir W. J. Herschel ; Photograph from a portrait of Galileo by Sustermans, presented by Sir W. Huggins.

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*The Attraction of the Himalaya Mountains upon the Plumb-line in India.** By Major S. G. Burrard, R.E., Superintendent of the Trigonometrical Survey of India.

1. In a paper read in 1895 before the Royal Society † the late General Walker advocated the employment of "groups" of astronomical stations for the purpose of eliminating the effects of local attraction at a central station. In the same paper he explained the preponderance of northerly deflections that prevails throughout India by the hypothesis that local attraction is producing a southerly deflection at Kalianpur, *the station of reference of the Indian Survey.*‡ In 1871 Archdeacon Pratt had come to the same conclusion ; in his "Figure of the Earth" he argues that an excess of density must exist near Kalianpur, for "the deflection

* For further details see Professional Papers of the Survey of India Department.

† *Phil. Trans.* clxxxvi. 745.

‡ No absolute deflection is known for any place ; all deflections are relative to that at Kalianpur. The term "northerly deflection" denotes a deflection "more northerly than the deflection at Kalianpur."

at Kalianpur," he writes, "is considerably south." Shortly after the issue of General Walker's paper in 1896, Sir David Gill's work on the Geodetic Survey of South Africa was published, in which the adoption of "groups" of astronomical stations was strongly recommended. In consequence of the opinions expressed by these eminent writers, and in full accord with their views, we decided in 1898 to determine the actual effect of local attraction at Kalianpur by means of a complete "group."

2. By the spring of 1899 astronomical observations for both latitude and azimuth had been taken at Kalianpur, and at eight adjacent stations, four being situated at an average distance of nine miles and four at an average distance of thirty-five miles from Kalianpur. A discussion of the results of the observations showed that local attraction was causing a *northerly* deflection at Kalianpur. This unexpected issue created a dilemma; either General Walker had been mistaken in advocating "groups," or his assumption of a southerly deflection at Kalianpur had been incorrect.

3. A classification of the results of the latitude observations discloses two remarkable facts: First, that two-thirds of the southerly deflections observed in India are situated within a narrow zone, running from east to west between the parallels of 24° and 26° approximately. Second, that large northerly deflections prevail from east to west between the parallels of 24° and 18° approximately.

4. I attached at first great significance to the fact that the parallel of 24°, along which approximately the direction of the deflection changes sign, happens to be the parallel of Kalianpur, the station of reference of the survey. I calculated the respective effects of the Himalayan mass and of the deficiency of matter in the Indian Ocean at twelve different stations, and found that if the Himalayas exercised their normal attraction on the plumb-line throughout India, and the deficiency of matter in the Indian Ocean exerted a northerly repulsive force, the sign of the deflections would change along the parallel of 24°. But this explanation of the observed phenomena could not be maintained; the results of the longitude arcs observed across India from coast to coast showed that the deficiency of matter in the Indian Ocean was wholly compensated by submarine strata of excessive density. If the effect of the Indian Ocean is compensated in the prime vertical it must be compensated also in the meridian.

5. After this first failure I undertook an investigation to see whether the change in the sign of the deflection could be due to errors of the ellipsoid of reference; but, though such errors would produce a change of sign along the parallel of the station of origin, the investigation showed that no considerable modification of the Clarke spheroid was possible.

6. The difficulty with which we are now confronted is this: it is clear that Himalayan attraction is *not* compensated so completely as Archdeacon Pratt believed, large deflections of

35″, 37″, 38″ having been met with in sub-Himalayan regions since the Archdeacon's death; but if Himalayan attraction is affecting the plumb-lines in India, the deflections north of latitude 24° should be more northerly than the deflection at Kalianpur, and the deflections south of latitude 24° should be less northerly. As has been explained above the observed deflections north of Kalianpur are southerly, and the observed deflections south of Kalianpur are northerly. (The deflections cannot be regarded as accidental or as due to "local attraction.")

7. To illustrate the observed phenomena the following table is given in which typical stations only have been entered. The meridian of 77° 30′ has been selected, but the same alternation of signs is observed on other meridians.

TABLE I.

Station.	Lat.	Observed deflection.	
	° ′	″	
Dehra Dun ...	30 19	−38	Himalayan influences.
Kaliana ...	29 31	− 7	
Noh	27 51	0	Belt of southerly deflections.
Daiadhari ...	24 38	+ 1	
Kalianpur ...	24 7	station of reference.	
Ladi	23 8	− 5	Belt of large northerly deflections.
Badgaon	20 44	− 8	
Damargida ...	18 3	− 3	
Cape Comorin ...	8 9	+ 2	

Under the Himalayas at Dehra Dun we find deflections of 38″; within sixty miles at Kaliana they dwindle to 7″; and in latitude 27° the deflections are southerly; south of Kalianpur at Ladi and Badgaon we meet with a succession of northerly deflections, which gradually become smaller as we proceed southwards to Cape Comorin.

8. The large deflections at Dehra Dun have hitherto not received the attention they deserve; they have been brushed aside with the remark, "At the foot of the Himalayas."

If Himalayan attraction is capable of producing a deflection of 38″ at Dehra Dun, its effects must be felt at Cape Comorin; on the other hand, if Himalayan attraction exercises no effect on plumb-lines south of Kaliana, it cannot produce a deflection of 38″ at Dehra Dun.

If a hemisphere of lead existed immediately north of Dehra Dun it might produce a large deflection there, and yet exercise no appreciable effect at Kaliana. But in the case of mountains—and the Himalayas are no exception—the vertical dimension is always small, and the change in elevation always gradual, and no overwhelming excess of matter can exist in the immediate

vicinity of a station situated even at their foot. In the *Philosophical Transactions of the Royal Society*, vol. 186, p. 803, General Walker wrote:—"It is thus evident that the effect of the attraction of mountain masses on the plumb-line, which may be very large in the immediate vicinity of the mountains, will be reduced at a distance in greater proportion than is assigned by an incomplete application of the law of gravitation, because of the deficiency in the density of the strata under the mountains, which has not hitherto been allowed for. Eventually a point must be reached at which the positive attraction of the matter above will be cancelled by the negative attraction of the deficiency below, and then the mountain masses will have no influence on the plumb-line."

When, however, one comes to apply in practice the theory propounded in this extract, one finds that if the Himalayas are compensated by underlying deficiencies of matter, the compensating cause will affect the plumb-lines at *both near and distant* stations.

9. I have calculated the effect of the attraction of the Himalayan mass at five stations on the meridian of 77° 30′, viz. Dehra Dun, Kaliana, Kalianpur, Damargida, and Cape Comorin, and have interpolated its effects at other stations from a plotted curve. The results are as follows :—

TABLE II.

Station.	Lat.	Calculated deflection due to—			Differential deflection from Kalianpur.
		Himalayan ranges.	Tibetan Plateau.	Total.	
	° ′	″	″	″	″
Dehra Dun ...	30 19	19	53	72	−54
Kaliana ...	29 31	2	34	36	−18
Noh ...	27 51	1	27	28	−10
Daiadhari ...	24 38		20	20	− 2
Kalianpur ...	24 7		18	18	...
Ladi ...	23 8		16	16	+ 2
Badgaon ...	20 44		13	13	+ 5
Damargida ...	18 3		10	10	+ 8
Cape Comorin ...	8 9		3	3	+15

10. In the following table the results of observation and calculation are compared :—

TABLE III.

Station.	Differential Deflections.		Discrepancy.
	By Observation. =0.	By Calculation. =0.	=0−0.
Dehra Dun ...	−38	−54	−16
Kaliana ...	− 7	−18	−11
Noh	0	−10	−10

Station.	Differential Deflections. By Observation. =O.	By Calculation. =C.	Discrepancy. =C−O.
Daiadhari ...	+ 1	− 2	− 3
Kalianpur ...	...	...	...
Ladi	− 5	+ 2	+ 7
Badgaon ...	− 8	+ 5	+13
Damargida ...	− 3	+ 8	+11
Cape Comorin ...	+ 2	+15	+13

11. At stations immediately north of Kalianpur the deflection by theory is northerly, but the observed deflection southerly; at stations immediately south the observed deflection is northerly and the theoretical deflection southerly. There appears therefore to be some hidden cause which is tending to reduce the northerly deflections at northern stations and to increase the northerly deflections at southern stations; and I have been led to infer the existence of an underground chain of excessive density that crosses Central India for 1,000 miles, and which underlies Balasore, Mandla, Bhopal, and Jodhpore. The attraction of this chain at stations situated to the north of it is opposed to Himalayan attraction; at stations situated to the south of it the two attractions act in concert. The chain imposes a wave on the curve of Himalayan attraction, and masks the true effects of the latter. If the hypothesis of the chain is correct, then the change in the sign of the deflections that occurs approximately along the parallel of 24° is not due in any way to the fact that this is the parallel of the station of reference. The proximity of the station of reference to the line of the chain appears, indeed, to be accidental. The significance that I attached to it in the early stages of this investigation appears to have been undeserved.

12. If a line exists towards which all plumb-lines from both sides throughout its entire length are deflected, the conclusion that it overlies a source of attraction may seem obvious, but it is not obvious. The hidden chain of Central India is so large that its presence has escaped detection. The idea that deflections of the plumb-line are accidental and due to "*local* attractions" is general, and every unexplained deflection is involuntarily attributed to a cause hidden *in the immediate* vicinity. When on flat plains we meet with a southerly deflection in latitude 27° and a northerly deflection in latitude 18°, we habitually prefer to assume two separate *local* hidden causes, rather than ascribe both deflections to a distant central cause.

13. The position of the underground chain coincides in places with the Vindhyan Mountains; but these latter are insufficient to produce the effects attributed to the chain; moreover, the effects of the chain continue to be observed to the east and to the west after the Vindhyas have ceased to be visible.

14. One feature of the chain is worthy of note: its course

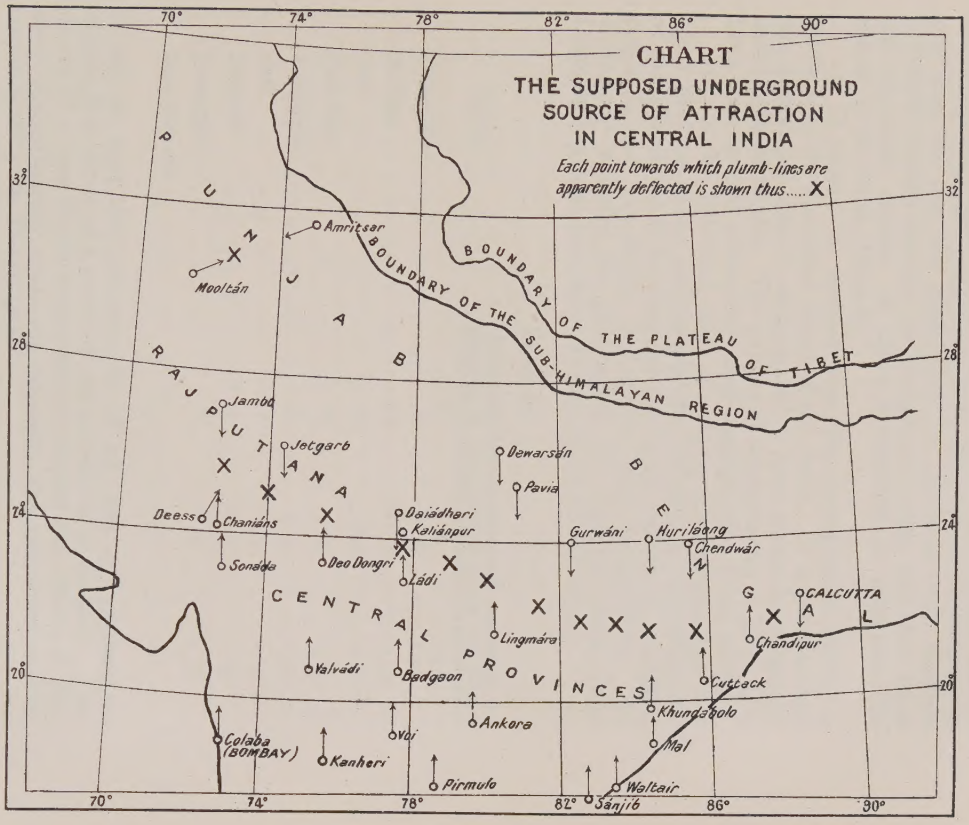
CHART
THE SUPPOSED UNDERGROUND SOURCE OF ATTRACTION IN CENTRAL INDIA
Each point towards which plumb-lines are apparently deflected is shown thus..... X
70°
74°
78°
82°
86°
90°
32°
28°
24°
20°
PUNJAB
RAJPUTANA
CENTRAL PROVINCES
BENGAL
Amritsar
Mooltán
Jamba
Jetgarh
Deessa
Chaniáns
Sonéda
Deo Dongri
Daiádhari
Kaliánpur
Ládi
Dewarsán
Pavia
Gurwáni
Huriláng
Chendwár
Lingmára
Yalvádi
Badgaon
Ankora
Vui
Kanheri
Colaba (BOMBAY)
Pirmulo
Waltair
Sánjib
Cuttack
Khundapolo
Mal
Chandipur
CALCUTTA
BOUNDARY OF THE SUB-HIMALAYAN REGION
BOUNDARY OF THE PLATEAU OF TIBET

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can be traced from longitude 88° to longitude 73°, and runs from E.S.E. to W.N.W., *in apparent parallelism with the Himalayas.* In the Punjab, to the *west* of the Himalaya Mountains, the longitude arc, Amritsar—Mooltan, reproduces the same extraordinary result as the latitude observations have given in Bengal to the *south* of the Himalayas. Just as the plumb-lines on all meridians from 88° to 73° seem to be deflected inwards towards one line, so are the plumb-lines at Amritsar and Mooltan deflected inwards towards the low-lying alluvium and away from the mountain masses. A chart is attached to illustrate the course of the chain (Plate 6).

15. It cannot be assumed that the discrepancies shown in Table III. are the actual effects of the chain. It is possible that the Himalayan mass is compensated to some extent by subjacent deficiencies of matter. The numerical discrepancies in Table III. have been derived without any such compensation being assumed.

16. In the following table are shown the discrepancies that occur between theory and observation if we assume the attraction of the Himalayas to be compensated to the extent of one-third, one-half, and two-thirds its normal force respectively. As Kalianpur is under the influence of the supposed chain, Cape Comorin has been now taken as the station of reference.

TABLE IV.

Station.	Discrepancies between theory and observation, the attraction of the Himalayan ranges and Tibetan plateau being assumed compensated to the extent of		
	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
Dehra Dun	− 6	+ 6	+17
Kaliana	−13	− 7	− 2
Noh	−15	−10	− 6
Daiadhari	−10	− 7	− 5
Kalianpur	− 8	− 5	− 3
Ladi	− 2	+ 1	+ 3
Badgaon	+ 3	+ 5	+ 7
Damargida	0	+ 2	+ 3
Cape Comorin	—	—	—

It will be seen that discrepancies of 13″ and 15″ continue to appear, if a theory of a one-third compensation be adopted. The theory of a two-thirds compensation renders the effects of the chain approximately equal and opposite, but introduces a discrepancy of 17″ at Dehra Dun.

17. In the following table are shown the discrepancies that occur between theory and observation if we assume the attraction of the successive Himalayan ranges to be uncompensated and that of the Tibetan plateau to be compensated to the extent of one-third, one-half, and two-thirds its normal force respectively :—

TABLE V.

Station.	Discrepancies between theory and observation, the attraction of the Tibetan plateau being assumed compensated to the extent of		
	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
Dehra Dun	−12	− 4	+4
Kaliana	−14	− 9	−4
Noh	−15	−11	−8
Daiadhari	−10	− 8	−5
Kalianpur	− 8	− 6	−3
Ladi	− 2	+ 1	+3
Badgaon...	+ 4	+ 5	+7
Damargida	+ 1	+ 2	+3
Cape Comorin	—	—	—

The hypothesis that the attraction of the outer Himalayan ranges are uncompensated, and that that of the Tibetan plateau is compensated to the extent of two-thirds its normal force, removes gross discrepancies and renders the effects of the chain at equidistant stations to the north and south of it equal and opposite. It makes the crest of the chain cut the meridian of 77° 30′ in latitude 23° 30′, and it attributes to the chain a maximum effect of 8″ at 200 miles north and south of its crest.*

18. This attempt to disentangle the effects of the chain from those of the Himalayas, and to obtain a definite numerical idea, is put forward more as an illustration of the data than as an exact solution of the problem. The hypothesis that the effects of Himalayan attraction are probably entangled with those of a chain running parallel to the Himalayas at an average distance of 400 miles affords, however, an explanation of the large deflections at Dehra Dun, of the southerly deflections north of latitude 24°, and of the northerly deflections south of latitude 24°.

On Periodic Orbits. By E. T. Whittaker.

The object of this note is to establish the results numbered (I) and (II) below: they relate to the theory of periodic solutions of the differential equations of dynamics, a theory which the researches of Hill, Poincaré, and Darwin have shown to be of great importance in dynamical astronomy.

Consider the motion of a particle in a plane, under the action of conservative forces. Let x, y be its coordinates at time t, referred to any fixed rectangular axes; its mass may be taken

* The discrepancies at the seven central stations are assumed to be due to the chain.

to be unity without affecting the generality of the investigation. Let $f(x, y)$ be its potential energy when at the point (x, y), so that its total energy may be taken to be

$$\tfrac{1}{2}\left(\frac{dx}{dt}\right)^2 + \tfrac{1}{2}\left(\frac{dy}{dt}\right)^2 + f(x, y).$$

The differential equations of motion of the particle form a system of the fourth order, and their general solution consequently involves four arbitrary constants. One of these constants is, however, merely the constant additive to t, which determines the epoch in the orbit, and so there are only ∞^3 really distinct orbits. This triple infinity of orbits can be arranged in sets, each containing a double infinity of orbits, by associating together those orbits which correspond to the same value of the constant of energy. We shall consider one such set of ∞^2 orbits, characterised by the condition that the constant of energy has the value h, so that

$$\tfrac{1}{2}\left(\frac{dx}{dt}\right)^2 + \tfrac{1}{2}\left(\frac{dy}{dt}\right)^2 = h - f(x, y).$$

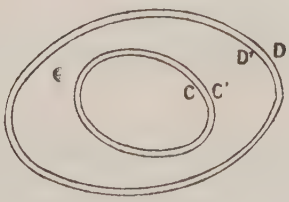
These orbits are defined analytically by means of the principle of least action; namely, that the orbit between two given points (x_0, y_0) and (x_1, y_1) is such as to make the value of the expression

$$\int \{h - f(x, y)\}^{\frac{1}{2}} \{(dx)^2 + (dy)^2\}^{\frac{1}{2}}$$

stationary as compared with other curves joining the given terminal points (x_0, y_0) and (x_1, y_1). Our attention will be directed particularly to those orbits which are *periodic*, i.e. which are in the form of closed curves.

1°. *A criterion which may be used in the discovery of periodic orbits.*

Consider any simple closed curve C in the plane of $x\,y$; and let another simple closed curve C' be drawn, enclosing C and



differing only slightly from it. We may regard C′ as defined by an equation of the form

$$\delta p = \phi(\gamma)$$

where δp is the normal distance between the curves C and C′, and γ is the inclination of this normal distance to the axis of x. Then if I be the value of the integral

$$\int \{h - f(x, y)\}^{\frac{1}{2}} \{(dx)^2 + (dy)^2\}^{\frac{1}{2}}$$

when the integration is taken round the curve C, and if $\mathrm{I} + \delta\mathrm{I}$ denote the value of the same integral when the integration is taken round the curve C′ (so that the symbol δ denotes an increment obtained in passing from C to C′), we have

$$\delta\mathrm{I} = \int \{(dx)^2 + (dy)^2\}^{\frac{1}{2}} \delta \{h - f(x, y)\}^{\frac{1}{2}} + \int \{h - f(x, y)\}^{\frac{1}{2}} \delta \{(dx)^2 + (dy)^2\}$$

But

$$\delta \{h - f(x, y)\}^{\frac{1}{2}} = -\tfrac{1}{2} \{h - f(x, y)\}^{-\frac{1}{2}} \left(\frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y \right)$$

$$= -\tfrac{1}{2} \{h - f(x, y)\}^{-\frac{1}{2}} \left(\frac{\partial f}{\partial x} \cos\gamma + \frac{\partial f}{\partial y} \sin\gamma \right) \delta p,$$

since $\delta x = \delta p \cos\gamma, \qquad \delta y = \delta p \sin\gamma.$

Also

$$\delta \{(dx)^2 + (dy)^2\}^{\frac{1}{2}} = \delta p \, . \, d\gamma$$

$$= \frac{\delta p}{\rho} \{(dx)^2 + (dy)^2\}^{\frac{1}{2}},$$

where ρ is the radius of curvature of the curve C at the point (x, y).

Thus

$$\delta\mathrm{I} = \int \delta p \{(dx)^2 + (dy)^2\}^{\frac{1}{2}} \{h - f(x, y)\}^{-\frac{1}{2}} \left\{ \frac{h - f(x, y)}{\rho} - \tfrac{1}{2} \cos\gamma \frac{\partial f}{\partial x} - \tfrac{1}{2} \sin\gamma \frac{\partial f}{\partial y} \right\}.$$

This equation shows that if the quantity

$$\frac{h - f(x, y)}{\rho} - \tfrac{1}{2} \cos\gamma \frac{\partial f}{\partial x} - \tfrac{1}{2} \sin\gamma \frac{\partial f}{\partial y}$$

is positive at all points of C, then $\delta\mathrm{I}$ is positive, and so the quantity I has its value increased when any curve surrounding C and adjacent to C is taken instead of C as the path of integration.

Now suppose that another simple closed curve D can be drawn enclosing C, and such that at all points of D the quantity

$$\frac{h - f(x, y)}{\rho} - \tfrac{1}{2} \left(\cos\gamma \frac{\partial f}{\partial x} + \sin\gamma \frac{\partial f}{\partial y} \right)$$

is negative. Then, in the same way, it can be shown that the quantity I is increased when any simple closed curve D', enclosed by D and adjacent to D, is taken instead of D as the path of integration.

When, therefore, we consider the aggregate of all simple closed curves situated in the ring-shaped space bounded by C and D—which is assumed to contain no singularity of the function $f(x, y)$—it is clear that the curve which furnishes the greatest value of I cannot be C or D, and cannot coincide with C or D for any part of its length. There exists, therefore, among the simple closed curves of this aggregate one or more curves K for which the value of I is greater than for all the other curves of the aggregate. Since K does not coincide with C or D along any part of its length, it follows that the curves adjacent to K are all members of the aggregate in question, and hence that the curve K furnishes a stationary value of I as compared with all the curves adjacent to it. The curve K is therefore an orbit in the dynamical problem. We have thus arrived at the theorem

I. *If one closed curve be enclosed by another closed curve, and if the quantity*

$$\frac{h-f(x, y)}{\rho}-\tfrac{1}{2}\cos\gamma\frac{\partial f}{\partial x}-\tfrac{1}{2}\sin\gamma\frac{\partial f}{\partial y}$$

be positive at all points of the inner curve and negative at all points of the outer curve, then in the ring-shaped space between the two curves there exists a periodic orbit of the dynamical problem.

This result may be regarded as an analogue, in the theory of periodic orbits, of those theorems in the theory of equations which furnish the position of the roots by criteria depending on the sign of expressions connected with the equation. As the quantity

$$\frac{h-f(x, y)}{\rho}-\tfrac{1}{2}\cos\gamma\frac{\partial f}{\partial x}-\tfrac{1}{2}\sin\gamma\frac{\partial f}{\partial y}$$

can be immediately calculated for every point on the curves C and D, depending as it does only on the potential-energy function and the curves themselves, this result furnishes a means of detecting the presence of periodic orbits. Thus if we take, for example, the potential-energy function to be that due to two Newtonian centres of force of intensity 50 at the points (1, 0) and (− 1, 0), so that

$$f(x, y)=-\frac{50}{\{(x-1)^2+y^2\}^{\frac{1}{2}}}-\frac{50}{\{(x+1)^2+y^2\}^{\frac{1}{2}}},$$

and take $h=-\frac{1}{2}$, and choose the curves C and D to be circles whose centre is at the origin and whose radii are 80 and 120 respectively, it is easily found that the conditions mentioned above are satisfied, and that the existence of a periodic orbit in the ring-shaped space between these circles can be inferred.

2°. *An integral connected with periodic orbits.*

We shall now suppose that the field of force in the dynamical problem considered is due to several centres of force, attracting according to the Newtonian law of the inverse square of the distance. We shall consider a periodic orbit, enclosing any number of these centres of force, and shall establish the following theorem :

The integral

$$\frac{1}{2\pi}\iint\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\{h-f(x,y)\}\right]dxdy,$$

where the integration is taken over the interior of any periodic orbit, is equal to the number of centres of force enclosed by the orbit, diminished by two.

Since the quantity $\{h-f(x, y)\}$ is infinite at each centre of force, it is necessary to discuss the legitimacy of integrating over a region which contains centres of force.

Suppose then that each centre of force is surrounded by a small circle having its centre at the centre of force. Let the periodic orbit be denoted by C, and let these small circles be denoted by $\gamma, \gamma', \gamma'', \ldots$ Then if S denote the region which is contained within C but exterior to $\gamma, \gamma', \gamma'', \ldots$ we have by the well-known theorem on the conversion of integrals taken over an area into integrals taken round the boundary of the area, that

$$\frac{1}{2\pi}\iint_{S}\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\{h-f(x,y)\}\right]dxdy$$

$$=\frac{1}{2\pi}\left[\int_C-\int_\gamma-\int_{\gamma'}-\int_{\gamma''}-\ldots\right]$$

$$\left[\frac{\partial}{\partial x}\log\{h-f(x,y)\}\,dy-\frac{\partial}{\partial y}\log\{h-f(x,y)\}\,dx\right]$$

where the integration on the left-hand side is taken over the region S, and the integrations on the right-hand side are to be taken in the counter-clockwise direction round the curves C, $\gamma, \gamma', \gamma'', \ldots$ respectively.

Now the integral previously denoted by I, when taken between any two points of a periodic orbit, must have a value that is stationary as compared with the integral taken along any adjacent path joining the two points; the quantity previously denoted by δI must therefore be zero when the integral is taken along the arc of a periodic orbit between any two points of it; and consequently, since the quantity δp which occurs in the expression for δI is arbitrary, we must have the equation

$$\frac{h-f(x,y)}{\rho}-\tfrac{1}{2}\cos\gamma\frac{\partial f}{\partial x}-\tfrac{1}{2}\sin\gamma\frac{\partial f}{\partial y}=0$$

satisfied at every point of a periodic orbit ; that is, we have the equation

$$\frac{2\{h-f(x,y)\}\{(dx)^2+(dy)^2\}^{\frac{1}{2}}}{\rho}=\frac{\partial f}{\partial x}dy-\frac{\partial f}{\partial y}dx$$

or

$$-2d\gamma=dy\frac{\partial}{\partial x}\log\{h-f(x,y)\}-dx\frac{\partial}{\partial y}\log\{h-f(x,y)\}$$

satisfied at every point of the curve C. We have therefore

$$\frac{1}{2\pi}\int_C\left[\frac{\partial}{\partial x}\log\{h-f(x,y)\}dy-\frac{\partial}{\partial y}\log\{h-f(x,y)\}dx\right]$$

$$=-\frac{1}{\pi}\int_C d\gamma=-\frac{2\pi}{\pi}$$

$$=-2.$$

We have next to consider the integrals which arise from the curves γ, γ', γ'' . . . We shall take first the integral arising from the curve γ, and, for simplicity, will suppose that the corresponding centre of force is at the origin, so that γ is a small circle with the origin as centre, and we can write

$$h-f(x,y)=\frac{\mu}{(x^2+y^2)^{\frac{1}{2}}}+\phi(x,y)$$

where μ is a constant and $\phi(x,y)$ is a function which is finite and continuous in the vicinity of the origin.

Then for points on γ we have, denoting the radius of γ by ρ,

$$\frac{\partial}{\partial x}\log\{h-f(x,y)\}=\frac{-\frac{\mu x}{\rho^3}+\phi_x(x,y)}{\frac{\mu}{\rho}+\phi(x,y)}$$

$$=-\frac{x}{\rho^2}+\text{terms which, when } \rho \text{ is small, are less important than this.}$$

Similarly

$$\frac{\partial}{\partial y}\log\{h-f(x,y)\}=-\frac{y}{\rho^2}+\text{less important terms.}$$

Thus

$$-\frac{1}{2\pi}\int_{\gamma}\left[\frac{\partial}{\partial x}\log\{h-f(x,y\}\,.\,dy-\frac{\partial}{\partial y}\log\{h-f(x,y)\}\,dx\right]$$

$$=\frac{1}{2\pi\rho^2}\int_{\gamma}(xdy-ydx)+\text{terms which may be neglected in comparison with these}$$

$$=\frac{2\pi\rho^2}{2\pi\rho^2}$$

$$=1.$$

Similarly each of the integrals arising from γ', γ'', . . . has the value 1.

Thus we have

$$\frac{1}{2\pi}\iint_{S}\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\{h-f(x,y)\}\right]dxdy$$

$$=-2+1+1+1+\ldots$$

the number of 1's on the right-hand side of the equation being equal to the number of centres of force enclosed by the periodic orbit C, and thus

$$\frac{1}{2\pi}\iint_{S}\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\{h-f(x,y)\}\right]dxdy$$

$$=\text{the number of centres of force enclosed by the periodic orbit, diminished by two.}$$

Now it appears from this that the result is the same whatever be the radii of the small circles γ, γ', γ'', . . . provided they are infinitesimally small. Thus we are entitled to speak of the integral of the quantity

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\{h-f(x,y)\}$$

taken over the interior of the contour C, because the arbitrary element of choice in the circles γ, γ', γ'', . . . does not affect the value of the integral. We thus obtain the result—

II. *The integral*

$$\frac{1}{2\pi}\iint\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\{h-f(x,y)\}\right]dxdy,$$

where the integration is taken over the interior of any periodic orbit, is equal to the number of centres of force enclosed by the orbit, diminished by two.

It may be observed that if the centres of force are not Newtonian, *i.e.* do not attract according to the law of the inverse square of the distance, the method which has been followed will lead to an analogous but slightly different result from that enunciated above.

Comparison of Photographic and Visual Magnitudes of the New Star in Perseus. By W. H. Robinson.

(*Communicated by Arthur A. Rambaut, M.A., Sc.D., F.R.S., Radcliffe Observer.*)

Observations of the brightness, spectrum, colour, position, motion, and nebulous envelope of *Nova Persei* have from time to time been published, but very few photographic magnitudes of the star have as yet appeared. The results given in the present paper may therefore be of some interest and value.

Visual observations of the new star were commenced at the Radcliffe Observatory on February 25, when the writer—on his own initiative—commenced a series of thirty-three photographs of the region of the *Nova*.

The instrument employed was a half-plate camera with 1-inch doublet of superior quality covering a field of about 30° diameter. The camera was mounted on a small equatorial stand, and was readily adjustable to any desired position. The images of the stars were recorded as trails, no clock-work being introduced. Ilford Special Rapid plates were employed, and the development was effected with pyro-gallic.

The photographic magnitudes given in the present paper have been obtained in the following manner :—

A list of comparison stars, fourteen in number, was first prepared (Table I.), care being taken to select, where possible, only such stars as were in the immediate vicinity of the *Nova*, and evenly distributed around it ; but in order to include the five brightest stars in this list it was found necessary to take in an area greater than that for the nine fainter ones, which are all contained within a field 3° by 5°.

The group of moderately bright stars near *a Persei* is clearly shown on most of the plates, but its zenith-distance is much less than that of the new star, and is in consequence unsuitable for determining magnitudes by the differential method. The inclusion of the stars comprising this group would have necessitated the application of absorption-corrections, more or less doubtful in amount ; and since the density of the trails of stars of any given magnitude increases with their declinations, further slight corrections depending on this cause would have been necessary. For these reasons this group has not been used and has not been included in the list.

The stars selected for comparison of magnitudes with the *Nova* were the following :—

TABLE I.

List of Comparison Stars.

Star's Name.	Tabular Mag. (Harvard Scale.)	Star's Name.	Tabular Mag. (Harvard Scale.)
Capella ...	... 0·18	*l* Persei	4·84
α Persei ...	... 1·94	30 Persei	5·37
β Persei ...	... 2·31	36 Persei	5·40
ε Persei ...	... 3·04	Arg. Z. + 45°, 811	5·64
δ Persei ...	... 3·18	Arg. Z. + 43°, 818	5·97
ν Persei ...	... 3·90	Arg. Z. + 44°, 734	6·23
κ Persei ...	... 4·01	Arg. Z. + 42°, 795	6·30

The trails of the comparison stars and *Nova* were examined in transmitted light with a focussing glass, and their densities estimated by steps of sequences on an arbitrary scale of approximately ten or fifteen steps to a magnitude, the brightest star on the plate, *Capella*, being taken as very nearly zero, and *ν Persei* as 50.

The portions of the trails selected for comparison were, with very few exceptions, those corresponding to the same G.M.T.'s of exposures. The reliability of these estimations is strengthened by the fact that not until the final stage of conversion from densities into magnitude had been completed was any reference made to magnitudes of the *Nova* from other sources.

When a series of estimations had been obtained in this way the results were laid aside, and a second examination of the plates, on the same lines, was independently carried out. A duplicate series was thus secured, the results of which exhibited such close agreement with those of the first that further measurements were considered unnecessary.

The two series of results are shown in the following table, each result being the mean of several estimations.

TABLE II.

Mean Photographic Scale Estimations of Brightness of Comparison Stars.

Star's Name.	1st Series.	2nd Series.	Adopted Mean.
Capella	4	3	3·5
β Persei	21	21	21
α Persei	23	21	22
ε Persei	29	28	28·5
δ Persei	32	32	32
ν Persei	50	50	50
κ Persei	60	59	59·5
l Persei	63	63	63

Star's Name.	1st Series.	2nd Series.	Adopted Mean.
30 Persei	66	66	66
Arg. Z. +45°, 811 ...	68	67·5	68
36 Persei	71	71	71
Arg. Z. +43°, 818 ...	80	81	80·5
Arg. Z. +44°, 734 ...	81	81·5	81
Arg. Z. +42°, 795 ...	82	82·5	82

The magnitude-equivalents for the arbitrary scale were next determined.

On square-ruled paper the scaled and tabular values were plotted, the former, from Table II., as ordinates, and the latter, from Table I., as abscissæ; the points thus obtained and a smooth curve drawn through them are exhibited in fig. 1.

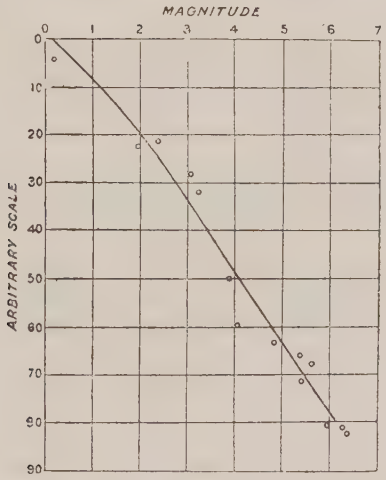


Fig. 1.—The position near 60 scale reading is that of κ Persei, an orange-coloured star.

From this curve the following equivalents were deduced :—

Table III.

Conversion of Scale into Magnitude.

Scale.	Magnitude Equivalent.	Scale.	Magnitude Equivalent.
0	0·15	30	2·75
5	0·65	35	3·10
10	1·15	40	3·45
15	1·60	45	3·75
20	2·00	50	4·05
25	2·40	55	4·40

Scale.	Magnitude Equivalent.	Scale.	Magnitude Equivalent.
60	4·75	75	5·80
65	5·10	80	6·15
70	5·45		

The equivalents given in Table III. were then adopted in forming the photographic magnitudes of the *Nova* shown in columns 5 and 6 of Table IV. A satisfactory agreement of the two sets of magnitudes is shown to exist. The greatest differences occur at the beginning of the series when the number of stars available for comparison was very limited and scattered over a comparatively large field, as before stated.

The second column of Table IV. contains the G.M.T. of exposure. The last column gives the visual magnitudes from Radcliffe observations, made at nearly the same G.M.T.'s, published in the *Monthly Notices* of the R.A.S., 1901 March, April, and May.

The general agreement of the last two columns—photographic and visual magnitudes of the *Nova*—is very striking. On two dates only, April 15 and 20, the differences of magnitude exceed 0·25, and in both these cases the photographic magnitude is *brighter* than the visual.

The eight bracketed results in column 7, independently derived from different photographs, are also remarkably accordant.

TABLE IV.

Comparison of Photographic and Visual Magnitudes of Nova Persei.

No.	G.M.T. 1901.	Photographic Scaled Densities. 1st Series.	Photographic Scaled Densities. 2nd Series.	Photographic Magnitudes. 1st Series.	Photographic Magnitudes. 2nd Series.	Photographic Magnitudes. Means.	Visual Magnitudes (Means).
1	Feb. 25 8^h	9	8·5	1·05	1·00	1·02	0·90
2	26 10	13	8	1·42	0·95	1·18	1·17
3	28 7	16	17	1·68	1·76	1·72	
4	9	18	16	1·84	1·68	1·76	1·58
5	March 1 8	23	21	2·24	2·08	2·16	1·99
6	5 7	27	24	2·54	2·32	2·43	2·42
7	9 9	46	44·5	3 83	3·72	3·77	
8	$9\frac{1}{2}$	45	45	3·75	3·75	3·75	3·61
9	11 11	45·5	46	3·78	3·81	3·80	3·93
10	12 11	44	44·5	3·69	3·72	3·71	3·87
11	13 8	44	47	3·69	3·87	3·78	4·03
12	16 8	46	46	3·81	3·81	3·81	
13	9	43·5	46·5	3·66	3·84	3·75	3·83
14	20 9	44	45	3·69	3·75	3·72	
15	$10\frac{1}{2}$	45	45	3·75	3·75	3·75	3·74
16	21 $9\frac{1}{2}$	48	47·5	3·93	3·90	3·92	
17	11	51	47·5	4·12	3·90	[4·01	3·91

No.	G.M.T.		Photographic Scaled Densities. 1st Series.	2nd Series.	Photographic Magnitudes. 1st Series.	2nd Series.	Means.	Visual Magnitudes (Means).
	1901.							
18	March 22	8h	63·5	65	5·00	5·10	5·05	5·20
19	23	9	45	42·5	3·75	3·60	3·68	3·92
20	25	8	65	65·5	5·10	5·13	5·12	5·15
21		11	65	66	5·10	5·17	5·13	
22	26	9	49	48·5	3·99	3·96	3·98	4·12
23	27	8	45	47	3·75	3·87	3·81	3·98
24		11	47	46	3·87	3·81	3·84	
25	28	9	61·5	59	4·85	4·68	4·76	4·61
26	30	9½	49·5	48	4·02	3·93	3·98	4·14
27		10½	50	49	4·05	3·99	4·02	
28	April 10	9	68	69	5·31	5·38	5·34	5·33
29	13	9	55·5	53	4·43	4·26	4·35	4·49
30	15	11	68	68	5·31	5·31	5·31	5·82
31	17	9	59	58	4·68	4·61	4·64	4·62
32	19	9½	67	70	5·24	5·45	5·34	5·13
33	20	9	71	71·5	5·52	5·56	5·54	5·83

No. 2. Not in good focus. Nos. 9 and 10. Very faint; very misty sky. No. 11. Images not good, too near edge of plate. No. 19. Varying images; cloudy. No. 30. Small altitude; trails good. No. 32. Very faint.

The last two columns of Table IV. are represented graphically in fig. 2. The ordinates of the two curves are, for the sake of clearness, separated by a constant quantity (one magnitude); unless this had been done the curves would have been superposed in places and would have presented a confused appearance.

A glance at this diagram will show that every change of

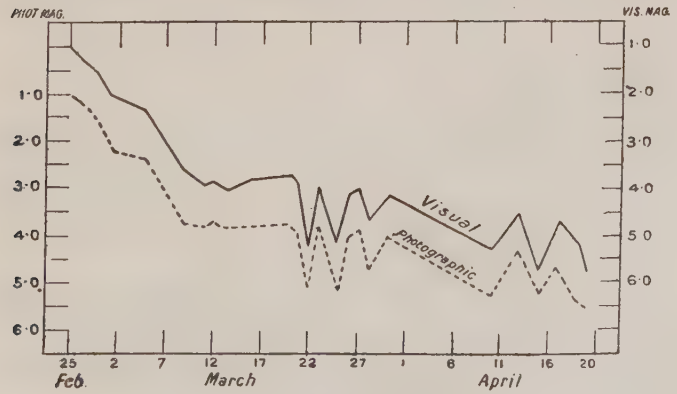


Fig. 2.—Comparison of Photographic with visual magnitude of Nova Persei.

brightness as determined visually has its counterpart in the photographic curve, so that the two corroborate each other in every particular.

The parallelism or identity of the curves in this figure appear the more remarkable when it is remembered that the *spectrum* of the star also exhibited considerable fluctuations in unison with those of magnitude, as proved by the extensive series of spectroscopic observations at Kensington and Stonyhurst. The director of the latter observatory, in a paper recently communicated to the *Astronomische Nachrichten* (No. 3756), says:—

"A comparison of all the photographic plates of the star's spectrum obtained at Stonyhurst with the light-changes observed at the Radcliffe Observatory, Oxford, has shown that the changes of the spectrum follow, not the phases of the light-curve, but the absolute magnitudes of the star's brightness. The new spectrum, which at first appeared to be associated with the minimum phase, was found on all the dates on which the star's magnitude was less than 4·57 both in March and April; and the old spectrum always returned when the magnitude was greater than 4·57."

Sir Norman Lockyer (*Proc. R.S.*, vol. lxviii. 231) writes:—

"Accompanying the great diminution in the light of the *Nova* observed on the evening of the 25th [March], the spectrum was found to have undergone a great change: the continuous spectrum had practically disappeared, and a line near D (probably helium, D_3) became more distinct. The other lines were hardly visible." The same astronomer further remarks:—"It has been noted that the lines 447, 501, and D_3 appear to vary with the magnitude of the star, becoming relatively more prominent towards a minimum" (*Proc. R.S.*, vol. lxviii. 403).

Closely related to the question of changes in the spectrum of the *Nova* is that of variation in the *visual colour*. On this subject Dr. Rambaut, in the *Monthly Notices* for 1901 April, wrote: "A comparison of the notes on the colour of the star with the variation in its brightness seems to indicate some relation between these two phenomena, and suggests that a diminution of light is accompanied by an increase in the redness of the star." In the next number of the *Notices* the Radcliffe Observer remarks:—"The suggestion made in the last paper that the change in tint was closely related to fluctuations in brightness has been fully confirmed." More recently Herr Osthoff has published (*Astronomische Nachrichten*, 3751) a diagram exhibiting the marked relationship which existed between the variations of visual magnitude and those of colour.

It would therefore appear that, notwithstanding the striking changes in the quality of the star's light, as shown by the spectroscope, and the changes observed in its colour, the variation in the general intensity of the light proceeded *pari passu*, whether measured visually or photographically.

The difficulty of obtaining magnitudes of the *Nova* from long-exposure photographs for comparison with visual magnitudes is

brought out clearly by Mr. H. Ellis in his paper entitled "Notes on some Photographs of *Nova Persei*," published in the *Journal of the British Astronomical Association*, vol. xi. No. 6. Mr. Ellis took his pictures of the *Nova* with a 5-inch portrait lens and with a large mirror on several nights from February 25 to March 10, using Ilford Special Rapid plates (carefully backed). He remarks :—"In examining these plates, it is very noticeable that the image of the *Nova* is large; in fact, it does not seem to have diminished in size, although by March 10 its light had much diminished and become very orange, with flashes of crimson, when viewed in the guiding telescope."

It will be seen in Table IV. of the present paper that a fall of nearly three magnitudes occurred between February 25 and about March 10.

Mr. Ellis proceeds : "In all the plates the image is larger than *α Persei*. . . . The reason of this, I think, must be the large gaseous envelope which the spectroscope shows to be present."

The nebulous envelope, if it existed at the time of Mr. Ellis' photographs, was not sufficiently actinic to be photographed with short exposures or by the trail method. The exposures by the trail method (within small limits of declination) possess the exceptional merit of being not only *short* but *always equal*; and these conditions seem to be essential when photographs of an object such as the *Nova* are required for comparison from night to night.

The chemically active quality of the light of the new star is emphasised by contrast with that of other reddish stars. This is shown on plate No. 7 (March 9, 9^h), which, after being exposed on the Nova, was exposed on the first magnitude orange-red star *α Orionis*, when it was at the same altitude as the new star and probably not so red.* From a discussion of the data afforded by this plate the resulting magnitudes were found to be :

	Photographic.	Visual.
α Orionis	3·20	0·91†
Nova Persei	3·77	3·61

It may be mentioned, in conclusion, that the trails were examined for any evidence of rapid changes of light similar to those seen on February 24 by Herr Archenhold,‡ but no sudden changes were found, except such as were attributable to atmospheric causes.

Radcliffe Observatory, Oxford: 1902 *January* 6.

* *Nova röthlich.* Harkányi (*A.N.* 3706). *Nova tief roth.* Köhl (*A.N.* 3709).

† Harvard Photometry.

‡ "Die plötzliche Lichtabnahme der Nova und ihr völliges Verschwinden waren so etwas Aussergewöhnliches, dass ich zunächst an eine atmosphärische Ursache dachte. Nur der Umstand, dass ich die umliegenden schwachen Sterne wie *ν*, *ψ*, *δ*, und *σ* Persei während dieser Zeit deutlich wahrnahm, liessen auch die Möglichkeit zu, dass es eine wirkliche Lichtabnahme der Nova sei" (*Ast. Nach.* No. 3694).

Period and Light Curve of 6685 *Y Lyræ.**

By A. Stanley Williams.

The earlier observations of this star seemed to suggest a moderately long period, and one of about eighty-six days was proposed by Dr. Hartwig in 1900,† whilst at about the same time I thought it might be forty-two days, or about half the above value. Observations of the star were commenced here in 1901 May with the object of solving this question, and after these had proceeded for some time it became evident that the real period was actually a very short one. The question was finally decided on August 18, on which date the star was observed shortly before the rise from minimum had commenced, and followed up to and past maximum ; whilst further observations soon showed that the true period was very nearly half a day. About the same time Dr. Hartwig was making a series of observations of the star at the Bamberg Observatory. His earlier observations are referred to in *A.N.* 3744.‡ These were inconclusive, but further observations in the latter part of August made him suspect that the variable had a short period. Dr. Hartwig was unable to continue his observations at this time, but later on he was able to fully confirm the rapid variability of the star, and his observations are in good agreement with the others.

The results contained in the present paper are based upon a total of 239 visual observations. Fourteen of these were made by Dr. Hartwig with the 10-inch Bamberg refractor. The others were made at Hove with a $6\frac{1}{2}$-inch reflector, a power of 110 being usually employed, but occasionally one of 225 was made use of on moonlight or hazy nights. These observations are all included in the period comprised between 1901 May 13 and November 3. The following is a short description of the method of observation, the reduction of the observations, and the determination of the magnitudes of the comparison stars.

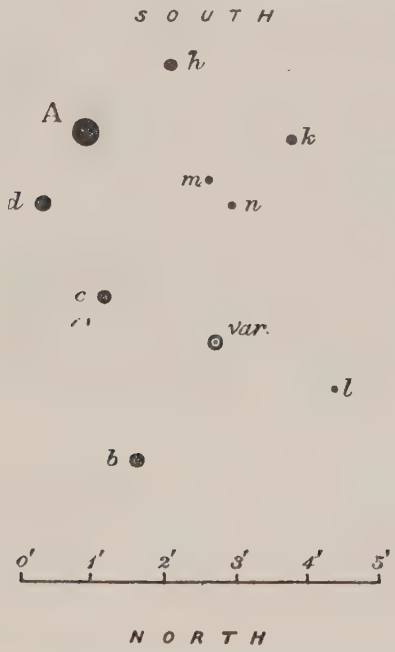
In order to eliminate the effects of what has been called "position error," three principal methods may be employed, namely (1) we may observe the group of stars under all possible angles of position with respect to a line through the eyes; (2) we may observe the group with the eyes always in the same position ; and (3) in comparing each comparison star separately

* The star is designated *X Lyræ* in the "Companion to the Observatory." Its place for 1900 is α 18^h 34^m 12^s, $\delta + 43^\circ$ $51'{\cdot}8$.

† *Vierteljahrsschrift d. Astr. Gesell.*, Jahrg. 35, Heft 4, p. 276.

‡ In this note doubt is thrown upon the identity of the variable with a star of about 12^m, but they are actually one and the same object. It will appear from the present investigation that the light curve is rather a peculiar one, and that the star is only at all bright for about three hours out of twelve, whilst the period is so nearly half a day that it is possible to observe the star night after night for a long interval of time without any decided change of brightness becoming apparent.

with the variable we may always observe with the eyes parallel to, or in some fixed position with respect to, a line joining the two stars observed. The first of these methods will undoubtedly give the best results, since with it the effects of position error must be altogether eliminated. Its only drawback is that the time and labour required are both much increased—a serious matter, especially in the case of a faint and rapidly varying star.* Method (2) was made use of in the present series of observations.† The group was always observed so that a line through the stars A and *b* (see diagram below) was normal to a line through the eyes, *b* being above. By this method the effects due to position error are not eliminated, so far as regards the absolute magnitude of the star observed, but the observations will be strictly comparable *inter se*.‡



* In connection with this method and the question of position error generally reference should be made to the valuable papers by Dr. A. W. Roberts in the *Monthly Notices*, vol. lvii. p. 483, and vol. lix. p. 524.

† These remarks relate only to the Hove observations except where otherwise mentioned. Method (3) seems preferable on the whole to method (2), and is the one at present made use of by the writer. The reflector with rotating tube is very convenient for either method.

‡ The effect of a complete reversal of position is to produce a change of about half a magnitude in the relative brightness of two stars observed in a direction normal to a line through the eyes by the writer.

The comparison stars used are shown in the preceding diagram.* The observations were made by the method of Argelander, and the step values of the different comparison stars, as derived from the very numerous comparisons between the latter and the variable, are as under :—

Comp. Star.	Step Value.	Adopted Mag.
c	0·0	10·70
h	5·5	11·09
k	13·0	11·63
m	14·8	11·75
n	18·1	11·99
l	19·7	12·10

The magnitude of the star *c* was assumed to be 10·7, chiefly from its visibility in a $2\frac{3}{4}$-inch refractor. On a clear night it is just distinctly and *steadily* visible, with a power of 75 on this instrument. The $6\frac{1}{2}$-inch reflector has been considered to show distinctly and steadily stars down to $12^{m}\cdot5$ on clear nights with a power of 110. The comparison star *l* is near, though decidedly above, this limit, and its magnitude has been assumed to be 12·1. Assuming the foregoing values for *c* and *l*, the magnitudes of the other comparison stars, as determined from the step values, are given in the last column of the above table, the value of a step being $0^{m}\cdot071$. With reference to the comparison stars, it may be mentioned that *c*, *h*, and *l* have always appeared steady in brightness, and the comparisons made with them were usually satisfactory and consistent, particularly those with *l*. The other three stars *k*, *m*, and *n* have proved more unsteady, so that the comparisons with them were seldom quite satisfactory : *k* in particular has fluctuated so much as to give rise to the suspicion that it may be variable.

A somewhat elaborate system of weighting has been used in reducing the observations. The method of Argelander is capable of yielding most accurate results so long as the difference in brightness of the two stars compared does not exceed a few steps ; but in proportion as such difference exceeds, say, 3 or 4 steps the result becomes less and less reliable. The following weights were finally adopted :—

	Weight
Comparisons with *l* not exceeding 3 steps	3
Comparisons with *l* above 3, but not exceeding 7 steps	2
Comparisons with other stars not exceeding 3 steps ...	2
Comparisons with *l* exceeding 7 steps	1
Comparisons with other stars exceeding 3 steps ...	1

* The positions of *h*, *k*, *m*, and *n* are only roughly estimated, but will be near enough to identify the stars. The bright star A is B.D.+43°·3029 (8·1 mag.).

The comparisons with *l* were given increased weight because this star proved more satisfactory than any of the others. All the observations were reduced with the above weights, excepting that in a few cases less weights were assigned when the notes made at the time of observation seemed to show that this was necessary.

From a preliminary determination from 5 maxima observed between 1901 August 19 and October 20 the period of variation was found to be approximately $12^h\ 3^m\ 40^s{\cdot}6$. For the final determination of this element, however, photographic observations have been used, chiefly on account of the much greater length of time available. Two photographs, dated 1899 December 31 and 1900 September 3, show the variable exactly of the same relative brightness with respect to *c*, in both the two stars appearing as a double star with slightly unequal components.* These photographs were taken with a 4·4-inch portrait lens of about 20 inches equivalent focus, and a third one obtained with the same instrument on 1901 November 5 also shows the variable of the same brightness with respect to *c*. In the case of photographs of a rapidly varying star there is necessarily some uncertainty as regards the exact time corresponding to the image of the star on the plate. As the exposures are not very dissimilar,† it has been assumed in the present instance that this time corresponds to the middle of the exposures. As we are now only concerned with the period of variation, and not with the time of maximum, it is immaterial whether this assumption is correct or not, provided that this time corresponds to the same phase in all the photographs. Below are given the date and heliocentric Greenwich mean time of the exposures and the duration of the latter. The fifth column contains the interval in time by which the observations follow the adopted epoch of T_o ‡, and the last column the difference from the mean.

Plate.	Date.	Time. h m	Exposure. m	Interval from T_o. h m	Diff. from Mean. m
46	1899 Dec. 31	6 40·4	63	0 54·2	−0·6
63	1900 Sept. 3	14 19·3	45	0 56·7	+1·9
149	1901 Nov. 5	9 9·4	32	0 53·6	−1·2

From these three observations the period of variation is :—

$$0^d{\cdot}5026876 = 12^h\ 3^m\ 52^s{\cdot}21$$

The remarkably close accordance of the three observations is no

* The photographic brightness of the variable is the same as the visual.

† Plate 46 was exposed longer than the others, but the night was slightly hazy, particularly towards the end, and this would tend to equalise matters.

‡ The time T_o, as explained further on, is the time when the variable attains to equality with the comparison star *l* in its rapid rise from minimum to maximum.

doubt to some extent an accident, but the star varies so rapidly near the time of maximum that I cannot think that any one of them can be in error by as much as 10^m, and the above period cannot be far wrong owing to the considerable interval of time covered by the observations, 1,341 periods being included between the first and last. It also satisfies all the visual observations.

Since the rise from minimum to maximum occurs with great rapidity, the instant when the variable attains to equality with the comparison star *l* has been adopted as the principal epoch. This instant is designated by T_o. There are twelve nights on which the variable was observed from minimum to maximum, and on eleven of these the maxima were also completely observed. The observations of each of these twelve nights were plotted separately upon squared paper, and a light curve drawn. From these light curves the times of T_o, and also the times of maximum, were determined by measurement. No mean light curve was made use of in this part of the work, but the results for each night were derived separately from the observations of that night. The reason for this was, chiefly, that some of the maxima seem to have been less acute than others, and I am not satisfied that this is all due to errors of observation. But though the maxima differ slightly in this way, the rise from minimum to near maximum is in every case represented by a straight line, so that the time T_o can be very exactly determined. Table I., below, contains the dates and heliocentric Greenwich mean times of T_o. The mean of these times, corresponding to 1901 September 4, $12^h\ 20^m$, has been adopted as the principal epoch.

TABLE I.

Observed Times of T_o.

No.	Date. 1901.	Observed Time. h m	Computed Time. h m	O−C. m
1	Aug. 18	10 24·4	10 8·7	+15·7
2	19	10 12·3	10 16·5	− 4·2
3	20	10 12·3	10 24·2	−11·9
4	21	10 32·3	10 31·9	+ 0·4
5	22	10 42·2	10 39·7	+ 2·5
6	23	10 43·2	10 47·4	− 4·2
7	24	10 52·2	10 55·2	− 3·0
8	26	11 13·1	11 10·6	+ 2·5
9	Sept. 4	12 5·3	12 20·3	−15·0
10	Oct. 20	6 10·3	6 12·5	− 2·2
11	25	7 1·0	6 51·2	+ 9·8
12	Nov. 3	8 10·5	8 0·9	+ 9·6

The greater accordance of observations 4, 5, 6, 7, and 8 is probably due to the observer having acquired a settled manner

of observing and to the favourable weather prevailing at that time. No. 11 is from observations by Dr. Hartwig, and the residual here includes any error in the reduction of his observations to the scale of the others. With regard to this point there is a little uncertainty, since the Bamberg observations, made with a refractor, seem to make the star a little brighter than those made with the reflector.*

Table II., below, contains the observed times of maximum, with the intervals by which these follow the observed times of T_o.

TABLE II.

Observed Maxima.†

Date.	Greenwich M.T. h m	Interval from T_o. h m
1901 Aug. 18	11 51	1 29
19	11 32	1 22
20	11 34	1 24
21	11 33	1 3
22	11 33	0 53
23	11 45	1 4
24	12 13	1 23
26	12 11	1 0
Sept. 4	13 32	1 25
Oct. 20	7 10	0 59
Nov. 3	9 12	1 0

The mean of the times in the last column is 71^m, but from the mean light curve the interval is 60^m. The latter has been adopted as the true interval. The separate results vary from 53^m to 89^m, a difference which seems too great to be ascribed wholly to errors of observation.

The reduced observations of magnitude, having been arranged in order of distance from the time T_o, were formed into groups.

* Dr. Hartwig's observations are given below. The last column contains the magnitude according to the adopted scale.

	Greenwich M.T. h m	m	Mag.
1901 Oct. 25	7 1	*c* 0·74 Y	12·08
	7 12	*c* 0·7 Y	12·03
	7 23	*c* 0·4 Y $0^m·35$ *l*	11·46
	7 33	*c* 0·25 Y $0^m·5$ *l*	11·17
	7 41	*c* 0·15 Y	10·98
	7 57	*c* 0·15 Y $0^m·6$ *l*	10·98

The night was not favourable, and cloud stopped observation after $7^h\ 57^m$. The rapid rise from near minimum to maximum is, however, very well shown, and, as in my observations, is represented by a straight line.

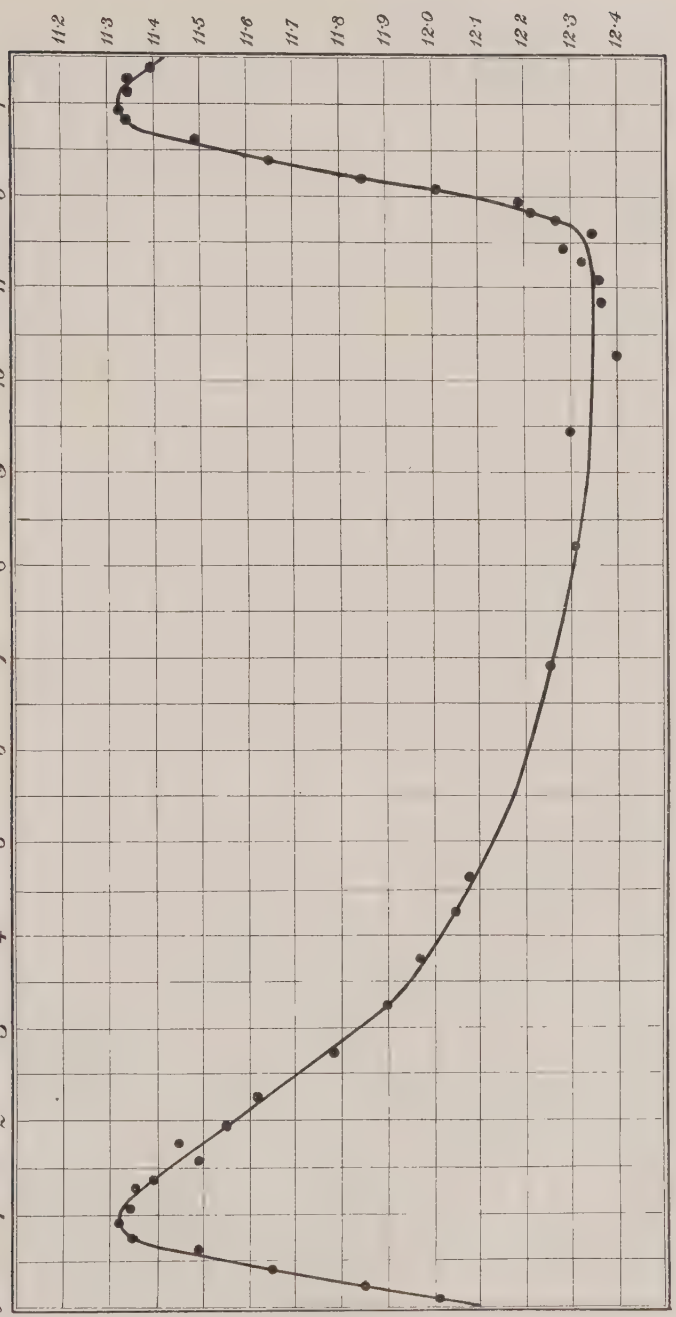
† Not reduced to the Sun.

For one hour before and two hours after T_o the observations have been collected into 10-minute groups. The interval of 14^m preceding this epoch was divided into two equal groups. For the remaining portion of the period the observations have been formed into half-hourly groups, excepting between 5^h and 10^h after T_o, where they are not numerous enough to permit of this. Table III. contains the results of this grouping.

TABLE III.

Grouped Observations.

No.	Interval from T_o. h m	Observed Magnitude.	Magnitude from Curve.	O−C. m	Weight.	No. of Obs.
1	0 4	12·01	12·01	0·00	37	12
2	0 13	11·85	11·85	·00	48	10
3	0 25	11·65	11·65	·00	54	13
4	0 35	11·49	11·48	+ ·01	37	11
5	0 45	11·35	11·35	·00	43	11
6	0 53	1·33	11·33	·00	38	9
7	1 5	11·35	11·33	+ ·02	38	9
8	1 17	11·35	11·36	− ·01	14	4
9	1 24	11·39	11·39	·00	32	8
10	1 34	11·49	11·44	+ ·05	19	4
11	1 44	11·44	11·48	− ·04	33	9
12	1 56	11·54	11·54	·00	17	5
13	2 14	11·62	11·63	− ·01	60	15
14	2 43	11·78	11·77	+ ·01	55	12
15	3 15	11·90	11·90	·00	59	13
16	3 42	11·96	11·97	− ·01	30	8
17	4 16	12·04	12·04	·00	49	10
18	4 42	12·08	12·09	− ·01	11	3
19	6 54	12·26	12·26	·00	6	2
20	8 18	12·31	12·32	− ·01	27	9
21	9 27	12·30	12·34	− ·04	15	5
22	10 13	12·40	12·35	+ ·05	16	7
23	10 47	12·37	12·35	+ ·02	18	8
24	11 4	12·36	12·34	+ ·02	16	7
25	11 16	12·33	12·33	·00	16	5
26	11 25	12·28	12·33	− ·05	14	5
27	11 36	12·35	12·32	+ ·03	13	6
28	11 47	12·27	12·27	·00	21	7
29	11 53	12·22	12·22	·00	17	6
30	11 59	12·19	12·17	+ ·02	18	6



Light Curve of Y Lyræ.

In the above table col. 1 contains the number of the group; col. 2, the interval from T_o; col. 3, the observed magnitude, this being the weighted mean of the observations forming the group; col. 4, the mean magnitude, measured from the mean light curve; col. 5, the residual, observed − curve; col. 6, the weight, this being merely the sum of the weights of the separate observations; and col. 7, the number of observations to the group.* The 30 observed magnitudes having been plotted upon a large scale diagram, the mean light curve was carefully drawn, and the magnitudes in the 4th col. were measured from this. This light curve, on a smaller scale, is reproduced in Plate 7. The observations are in very satisfactory accordance, the average deviation of a group from the mean light curve being only 0·014 mag., whilst no residual exceeds 0·05 mag. It should be mentioned, moreover, that the observations forming a group are also generally very accordant, and that no observation has been rejected for discordance. The only observations not used consist of a few in which the variable was not seen, and was accordingly noted only as being fainter than a certain magnitude.

It will be seen that the mean light curve presents some rather peculiar features. The rise from minimum to maximum is very rapid; whilst immediately after the latter is attained the decline sets in, and the fall, though not so rapid as the rise, is yet very quick for the first two hours after maximum. One result of this is that the variable is only at all bright for about three hours out of the twelve, and for the remaining nine hours it is more or less faint. The light curve has thus a very close resemblance to that of what has been termed the "cluster type" of variable. This type has been recently described very fully in an interesting article by Professor S. I. Bailey,† in which the approximate light curves from photographic observations of eight of the variables in the cluster Messier 5 are reproduced. Reference should also be made here to a paper by Professor E. E. Barnard ‡ on visual observations of some of the variables in the same cluster, and to one by Professor E. C. Pickering § on the rate of increase of the light of three variable stars in the cluster Messier 3. In length of period also the present variable star bears a close resemblance to those of the cluster type, the average period of forty variables in Messier 5 being $12^h\ 37^m{\cdot}4$ according to Bailey, whilst the periods of the three in Messier 3 investigated by Pickering are stated to be $12^h\ 12^m\ 25^s$, $12^h\ 0^m\ 15^s$, and $12^h\ 24^m\ 31^s$.

Although the presence of so many variables of the type in star clusters would seem to show that such a condition is

* Each of these separate observations is itself generally based on comparisons with from two to four comparison stars.

† *Astrophysical Journal*, vol. x. pp. 255–260.

‡ *A.N.* 3519.

§ *Astrophysical Journal*, vol. xii. p. 159. See also a valuable paper by the same writer on variable stars in clusters in *A.N.* 3525.

peculiarly liable to give rise to this kind of variation, yet the existence of isolated stars, like *Y Lyræ*, belonging to the same class would indicate that it is not an essential condition. In other words the type is not peculiar to clusters.

Elements of Y Lyræ.

Period of variation	$0^d \cdot 5026876 = 12^h\ 3^m\ 52^s \cdot 21$.
Epoch of T_o	1901 Sept. 4, $12^h\ 20^m$ G.M.T. (J.D. 2415632·5138.)
Epoch of maximum	1901 Sept. 4, $13^h\ 20^m$ G.M.T. (J.D. 2415632·5554.)
Maximum brightness ...	$11^m \cdot 32$.
Minimum „ ...	$12^m \cdot 35$.*
Minimum to maximum ...	$1^h\ 40^m$.†
Maximum to minimum ...	$10^h\ 24^m$.
Ratio increase to decrease ...	0·16.

Computed Times of T_o.‡

1902.	G.M.T. h m	1902.	G.M.T. h m
April 1	3 6	Aug. 9	19 52
11	4 23	19	21 10
21	5 41	29	22 27
May 1	6 58	Sept. 8	23 44
11	8 16	18	0 54
21	9 33	28	2 11
31	10 50	Oct. 8	3 29
June 10	12 8	18	4 46
20	13 25	28	6 4
30	14 43	Nov. 7	7 21
July 10	16 0	17	8 38
20	17 17	27	9 56
30	18 35		

Hove: 1902 *January* 4.

* According to Dr. Hartwig, the actual range of variation is somewhat less than a magnitude.

† It has been assumed that the minimum occurs just before the rise commences, but the star actually remains practically stationary for two or three hours before the commencement of the rise.

‡ The maxima will follow these times by one hour.

Σ 1639. 68 *Comæ Berenicis.* By Thomas Lewis.

R.A.	$12^h\ 19^m\ 25^s$	1900	Mag. A 6·7
N.P.D.	63° 52′		Mag. B 7·9

The measures commence in 1827 by W. Struve ; but they are few in number owing to the closeness of the components since 1880. It is undoubtedly binary, and as the distance is now increasing again it will become a fairly easy object and is worth attention.

Of course the orbit here given is only approximate, and is intended merely to call attention to the pair.

The annual means of the measures are :—

Year.	Angle.	Distance.	
1827·28	291°·2	1″·27	Σ (1)
29·39	289·7	1·15	Σ (3)
36·49	292·8	1·18	Σ
41·39	289·7	1·03	OΣ (1)
42·37	285·5	1·19	OΣ (2)
43·33	293·7	0·98	Maed. (1)
44·32	289·2	1·18	OΣ (1) Maed. (1)
48·35	288·8	0·96	Maed. (1)
55·32	282·2	0·97	OΣ (1)
56·48	288·1	0·87	Secchi (1)
57·33	283·6	0·84	Secchi (1)
58·34	283·0	1·00	De (4)
63·16	285·7	0·67	De (4)
70·31	275·8	0·74	OΣ (1)
75·25	273·6	0·40	De (1) Schiap. (4)
76·46	272·6	0·39	De (1)
77·41	271·4	0·40	De (1) Schiap. (4)
78·31	268·3	0·36	De (1)
79·31	268·7	0·4	Schiap. (2)
81·37	260·7	0·4	Schiap. (2)
82·39	258·1	0·39	Schiap. (4)
83·30	259·5	0·34	Engel (16) Schiap. (3)
84·37	243·7	0·38	Schiap. (2)
85·41	249·0 (?)	...	Schiap. (2)
87·39	236·0	0·22	Schiap.
89·40	215·6	0·2	H. Struve (1)
90·46	Single		H. Struve

Year.	Angle.	Distance.	
	°	"	
92·36	Single		Burnham (36-inch)
93	Single		Lewis (13-inch)
95	Single		Lewis (28-inch)
95·24	97·1	0·28	Lewis (1)
95·32	25·4	0·2	H. Struve (3)
96·34	19·3	0·15	Lewis (1)
97·26	12·7	0·25	Aitken (3) Lewis (2)
98·30	9·7	0·27	Aitken (3) Lewis (1)
99·37	3·0	0·26	Aitken (3) Lewis (3)
00·32	356·9	0·19	Bowyer (3) Lewis (3)
01·28	356·7	0·32	Aitken (3) Lewis (3) Bryant (1)

If from these measures we adopt the following six positions by taking means of neighbouring years—

	°	"
1831·05	291·2	1·20
43·94	289·4	1·07
58·21	284·5	0·87
76·21	271·6	0·45
82·35	259·4	0·38
85·88	239·9	0·30

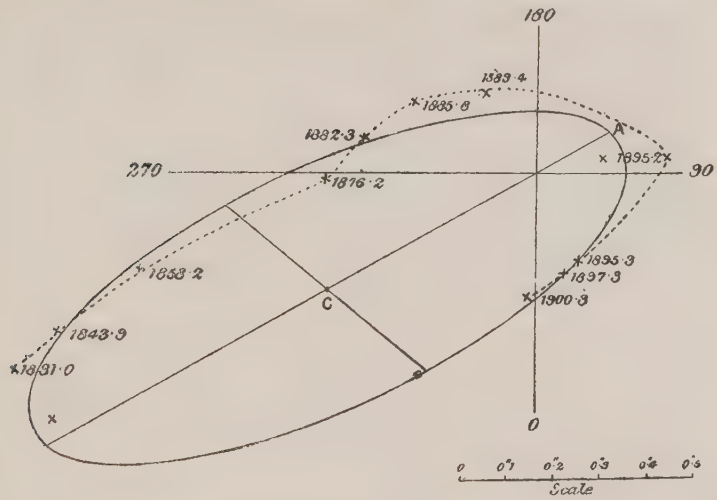
and plot them out, we find that so far as direction alone is concerned they are very closely represented by a straight line in the direction 123°·0; but the rate of motion along this line is

	"	
from 1831 to 1843	·092	in one year
1858	·126	,,
1876	·171	,,
1882	·173	,,
1885	·186	,,
1889	·210	,,

which clearly indicates orbital motion, and explains why the pair was not measurable in 1892 and 1893. It also shows that the recent measures are in the first quadrant.

We then have in addition to the above six normal places—

1895·24	97·1	0·28
1897·30	13·1	0·22
1900·32	358·9	0·26



These positions are shown in the figure which is the adopted apparent ellipse, from which the following elements for the true ellipse have been computed :—

$\tau = 1892{\cdot}0$		$\Omega = 109°\ 12'$	
$\rho = 180$ years		λ	18 7
$\epsilon = 0{\cdot}70$		γ	58 9
$a = 0''{\cdot}71$		ϕ	$42°{\cdot}88$

The measures of H. Struve were received after this paper was written. They uphold the binarity and conclusions here given in the most decided manner.

1901 *December* 28.

Observations of the Satellite of Neptune from Photographs taken at the Royal Observatory, Greenwich, in 1899–1900.

(*Communicated by the Astronomer Royal.*)

By inadvertence the publication of these results for the opposition of 1899–1900 has been delayed till now.

The photographs were taken with the 26-inch refractor of the Thompson equatorial. An occulting shutter immediately in front of the plate has been used to screen the planet during the greater part of the long exposure on the satellite, a series of very short exposures (usually twenty of one second each) being given to

Neptune at regular intervals (usually each minute) by lifting the occulting arm. The orientation was determined usually by means of a pair of short-exposure images of *Neptune*, the clock being put out of gear for seven seconds between the exposures to give a convenient displacement in R.A. The photographs were measured in reversed positions of the plate by each of two observers. The mean values of position angle and distance as measured are given in the following table, the tabular positions being computed from the data given in the *Connaissance des Temps*, based on Mr. H. Struve's elements, the eccentricity of the orbit being neglected owing to the uncertainty as to the present position of the periastron.

Positions of Neptune's Satellite measured on Photographs taken with the 26-inch Refractor.

Date.	Exposures.	Position angle.			Distance.		
		Observed.	Tabular.	Tab.−Obs.	Observed.	Tabular.	Tab.−Obs.
1899. d h m	m s.	°	°	°	″	″	″
Sept. 28 12 40	30 & 15	53·09	51·95	−1·14	14·37	14·41	+0·04
Oct. 10 12 3	30 & 20	42·67	39·57	−3·10	12·81	13·12	+0·31
12 12 11	30 & 10	261·23	261·88	+0·65	16·74	16·56	−0·18
16 11 40	30 & 10	32·27	32·55	+0·28	12·90	12·46	−0·44
1900.							
Jan. 17 10 2	20 & 10	(80·52)	83·08	(+2·56)	...	...	...
18 8 58	20 & 20	42·34	41·73	−0·61	14·45	13·81	−0·64
24 10 45	30 & 20	28·55	29·33	+0·78	12·35	12·49	+0·14

October 10. Image of planet elongated. January 17. Planet on edge of shutter.

Observations of Occultations of Stars by the Moon and Phenomena of Jupiter's Satellites, made at the Royal Observatory, Greenwich, in the Year 1901.

(*Communicated by the Astronomer-Royal.*)

Day. 1901.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
March 26	Disapp. 68 Orionis	Astrographic Equat.	225	Dark	9 13 58·04	W.
April 22	,, Lalande 11088	Great Equat. (Corbett)	100	,,	9 25 20·88	H. F.
22 (*a*) (*b*)	,, χ^2 Orionis	Astrographic Equat.	225	,,	10 25 0·03	R.
23 (*a*)	,, W. B. (2) VI. 1200	Old Altazimuth	100	,,	8 5 54·76	A. C.
23 (*a*)	,, ,,	Sheepshanks Equat.	55	,,	8 5 54·85	W. B.
23	,, ,,	Astrographic Equat.	225	,,	8 5 54·57	S.
23	Reapp. ,,	Sheepshanks Equat.	55	Bright	8 23 59·51	W. B.
28 (*b*)	Disapp. p^2 Leonis	Great Equat.	670	Dark	10 42 18·68	B.
May 31	,, Piazzi XV. 96	Sheepshanks Equat.	120	,,	11 48 44·94	A. C.
June 4	Reapp. Lalande 35497	,, ,,	120	,,	12 41 47·97	B.
29	Disapp. Lalande 30725	Astrographic Equat.	225	,,	8 45 47·44	H.
July 28 (*b*)	,, 21 Sagittarii	,, ,,	225	,,	9 50 35·54	R.
Aug. 24 (*c*)	,, Piazzi XVII. 323	,, ,,	225	,,	8 46 48·78	G. B.
Oct. 22 (*d*)	,, c^1 Capricorni			,,	No Occultation	A. C., R.
23 (*b*) (*c*)	,, κ Aquarii	Great Equat.	670	,,	8 52 47·81	B.
27 (*e*)	,, 29 Arietis	Astrographic Equat.	225	Bright	12 14 58·90	H.
27 (*f*)	Reapp. ,,	,, ,,	225	App. Bright	13 7 24·02	H.

Day. 1901.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Dec. 18	Disapp. λ Piscium	Great Equat.	670	Dark	7 23 57·86	B.
18	,, ,,	Sheepshanks Equat.	120	,,	7 23 57·80	W. B.
18	,, ,,	Great Equat. (Corbett)	120	,,	7 23 57·91	W. H. B.
18	Reapp. ,,	Great Equat.	670	Bright	8 7 40·59	B.
18 (*g*)	,, ,,	Sheepshanks Equat.	120	,,	8 7 (55)	W. B.

(*a*) The dark limb of the Moon was plainly visible.
(*b*) Instantaneous.
(*c*) Very faint; misty.
(*d*) This occultation was predicted in the *Nautical Almanac*, but did not actually occur, the star remaining visible throughout.
(*e*) Star appeared to glide up to and behind limb.
(*f*) Observer noted "possibly 0ˢ·5 late." The limb was apparently bright, the width of the dark lune being about 1″.
(*g*) The time was noted as "doubtful" by the observer, and is evidently erroneous.

Phenomena of Jupiter's Satellites.

Day. 1901.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation. h m s	Mean Solar Time of *N.A.* h m	Observer.
July 23	I.	Tr. Ing. First contact	Sheepshanks Equat.	100	9 35 6·29		H.
23	I.	Bisection	,, ,,	,,	9 40 5·48	9 43	,,
23	I.	Last contact	,, ,,	,,	9 44 14·79		,,
23	IV.	Occ. R. First seen	,, ,,	,,	9 44 34·74		,,
23	IV.	Bisection	,, ,,	,,	9 50 3 84	9 44	,,
23	IV.	Last contact	,, ,,	,,	9 58 32 43		,,
24	I. (*a*)	Ec. R. First seen	Astrographic Equat.	225	9 51 47·36	9 52	R.

Day.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation. h m s	Mean Solar Time of *N.A.* h m	Observer.
1901. July 27	II.	Occ. D. First contact	Astrographic Equat.	225	10 12 45·02		H.
27	II.	Bisection	,, ,,	,,	10 16 14·45	10 15	,,
27	II.	Last seen	,, ,,	,,	10 18 54·00		,,
Aug. 7	I. (*b*)	Occ. D. First contact	,, ,,	,,	10 28 51·56		,,
7	I.	Bisection	,, ,,	,,	10 31 41·10	10 35	,,
7	I.	Last seen	,, ,,	,,	10 36 30·31		,,

(*a*) Limb unsteady, cloudy.

(*b*) Limb rather diffused and tremulous, but the observation considered good.

The apertures of the telescopes used are as follows :—

	inches.		inches.
Great Equatorial	28	Great Equatorial (Corbett Telescope) ...	$6\frac{1}{2}$
Astrographic Equatorial (Guiding Telescope)	10	Old Altazimuth	4
Sheepshanks Equatorial	$6\frac{3}{4}$		

The initials H., A. C., B., W. B., R, H. F., W., S., G. B., W. H. B., are those of Mr. Hollis, Mr. Crommelin, Mr. Bryant, Mr. Bowyer, Mr. Rendell, Mr. Furner, Mr. Witchell, Mr. Showell, Mr. Bischlager, and Mr. Brookes respectively.

Royal Observatory, Greenwich : 1902 *January* 8.

Errata in Mr. Saunders' paper.

Vol. lx. page 189, lines 12, 21 (twice).

,, lxii. ,, 46, last line, and page 47, 8th line from bottom, *for* $(Dx-Ey)$ *read* $(Ex-Dy)$.